

Data Transmission Performance in the Presence of Carrier Phase Jitter and Gaussian Noise

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I. INTRODUCTION

In the operation of data transmission systems over voice-grade telephone channels, phase jitter¹ is a commonly observed transmission impairment. It appears in the form of low-index angle modulation of the received data signals. It is believed that phase jitter is a very important parameter in determining system performance. Therefore, many complicated methods^{2,3} have been developed to recover the jittered carrier. However, recent field measurements¹ show that the phase jitter in Bell System carrier systems has improved significantly over the past few years. As a result of this improvement, the following question naturally arises: How much phase jitter recovery is required for two-level and four-level systems?

In this B.S.T.J. Brief, we analyze the system performance degradation caused by phase jitter. The results suggest that for two-level systems, jitter need not be recovered and that for four-level systems, a coarse jitter recovery system would provide acceptable performance.

II. GENERAL CONSIDERATIONS

A simplified block diagram of a general VSB-AM data system is depicted in Fig. 1. A random message sequence $\{a_n\}$ is used to modulate an identically shaped pulse train, which is then transmitted over a voice-grade telephone line. The received random pulse train is corrupted by additive Gaussian noise and intersymbol interference;* the latter is slowly time-varying and is caused by the phase jitter in the carrier system which causes crosstalk between the in-phase and quadrature channels. The received pulse train is processed and sampled to provide the estimates of the transmitted message sequence $\{\hat{a}_n\}$. A useful measure of the performance of such a data system is the error probability, $P_e\{\hat{a}_n \neq a_n\}$.

III. THE PROBABILITY BOUND

If the peak-to-peak phase jitter is small, then the pulse train presented

* Channel amplitude and delay distortion are assumed to be removed by an equalizer and other impairments such as nonlinear distortion and impulse noise are neglected.

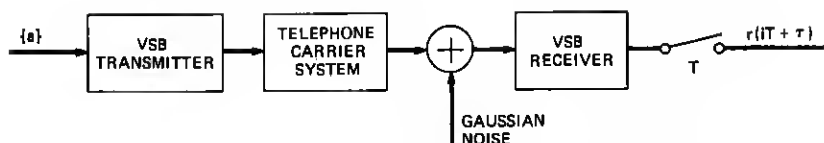


Fig. 1—Block diagram of a general VSB-AM system.

to the sampler at the receiver in the presence of additive Gaussian noise may be approximated by

$$r(t) = \sum_K a_K f(t - KT) + \phi(t) \sum_K a_K \bar{f}(t - KT) + n(t), \quad (1)$$

where the a_K are the transmitted symbols, $f(t)$ and $\bar{f}(t)$ are the basic in-phase and quadrature pulse response of the system, $1/T$ is the baud, $n(t)$ is a zero-mean Gaussian noise process and $\phi(t)$ is the phase jitter which is the sum of slowly varying sinusoids, $\phi(t) = \sum_i P_i \cos(\omega_i t + \theta_i)$. We assume the a_K are identically and independently distributed random variables with probabilities

$$P\{a_K = 2j + 1\} = \frac{1}{2M}, \quad j = -M, \dots, -1, 0, 1, \dots, (M - 1). \quad (2)$$

The probability density of $n(t)$ is

$$P(n(t)) = (2\pi\sigma_n^2)^{-1/2} \exp(-n^2(t)/2\sigma_n^2). \quad (3)$$

The m th transmitted symbol is determined by sampling $r(t)$ at $t = t_0 + mT$, i.e.,

$$\begin{aligned} r(t_0 + mT) = & a_m f(t_0) + \sum_{K \neq m} a_{K-m} f(t_0 - KT) \\ & + \phi(t_0 + mT) \sum_K a_{K-m} \bar{f}(t_0 - KT) + n(t_0 + mT). \end{aligned} \quad (4)$$

The error probability is defined to be

$$\begin{aligned} P_e = & P_r\{\hat{a}_m \neq a_m\} \\ = & \frac{2M-1}{M} P_r\left\{ \sum_{K \neq m} a_{K-m} f(t_0 - KT) + \phi(t_0 + mT) \right. \\ & \left. \cdot \sum_K a_{K-m} \bar{f}(t_0 - KT) + n(t_0 + mT) > f(t_0) \right\}. \end{aligned} \quad (5)$$

Assume the sampling instant is perfect and that there is no channel amplitude or delay distortion, i.e., $f(t_0 - KT) \equiv 0$ for all $K \neq 0$, and $f(t_0) = 1$. This is a reasonable assumption if an adaptive equalizer is

incorporated in the system. Thus, (5) can be rewritten as

$$P_e = \frac{2M-1}{M} P_r \{ n(t_0 + mT) + \phi(t_0 + mT) \cdot \sum_K a_{K-m} \tilde{f}(t_0 - KT) > f(t_0) \}. \quad (6)$$

Applying the Chernoff bound⁴

$$\Pr \{ z > y \} \leq \exp \{ -\lambda y \} \langle \exp (\lambda z) \rangle, \quad \text{all } \lambda > 0 \quad (7a)$$

and the following inequality

$$\langle \exp \{ a_K \cdot x \} \rangle \leq \exp \{ x^2 \cdot \sigma_a^2 / 2 \} = \exp \{ x^2 \cdot (2M-1)(2M+1)/6 \}, \quad (7b)$$

to (6), we obtain an upper bound on the conditional error probability

$$P_e |_{\phi(t_0+mT)} \leq \frac{2M-1}{M} \cdot \exp \left\{ - \frac{f^2(t_0)}{2(\sigma_n^2 + \phi^2(t_0 + mT)) \cdot \frac{(2M+1)(2M-1)}{3} \cdot \sum_K \tilde{f}^2(t_0 - KT)} \right\}. \quad (8)$$

From (8) it can be seen that an upper bound of the error probability is

$$P_e \leq \frac{2M-1}{M} \cdot \exp \left\{ - \frac{f^2(t_0)}{2(\sigma_n^2 + \phi_P^2) \cdot \frac{(2M+1)(2M-1)}{3} \cdot \sum_K \tilde{f}^2(t_0 - KT)} \right\}, \quad (9)$$

where ϕ_P is the maximum phase.

IV. EXAMPLE

A multilevel single-sideband AM system is used as a vehicle to determine the performance degradation caused by the phase jitter. The system signal-to-Gaussian noise ratio is assumed to be 24 dB and is defined as

$$S/N = \frac{\langle a_K^2 \rangle \cdot f^2(t_0)}{\sigma_n^2}. \quad (10)$$

The power of the signal is normalized to unity, i.e., $f^2(t_0) = 1$ and $\sum_K \tilde{f}^2(t_0 - KT) = 1$. Figures 2 and 3 are plots of the probabilities of error bound versus peak-to-peak phase jitter for two- and four-level

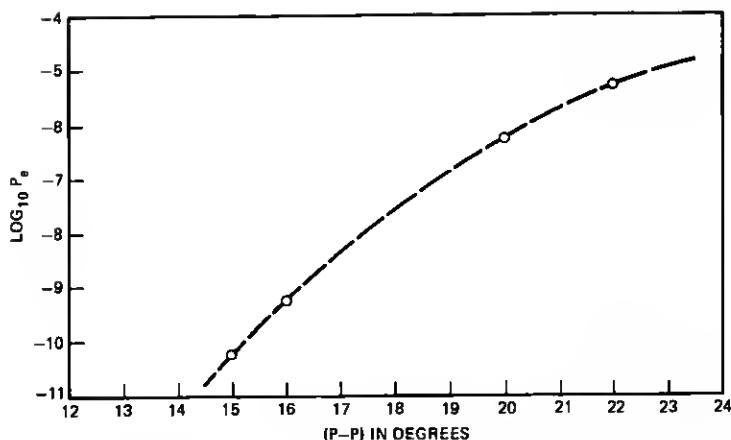


Fig. 2—The probability of error bound versus peak-to-peak phase jitter $f^2(t_0) \cdot \sigma_a^2 / \sigma^2 = 24$ dB, two-level system.

signaling. It can be seen from these curves that if the peak phase jitter for two- and four-level systems is limited to less than 20 and 6 degrees respectively, a probability of error less than 10^{-6} is achieved.

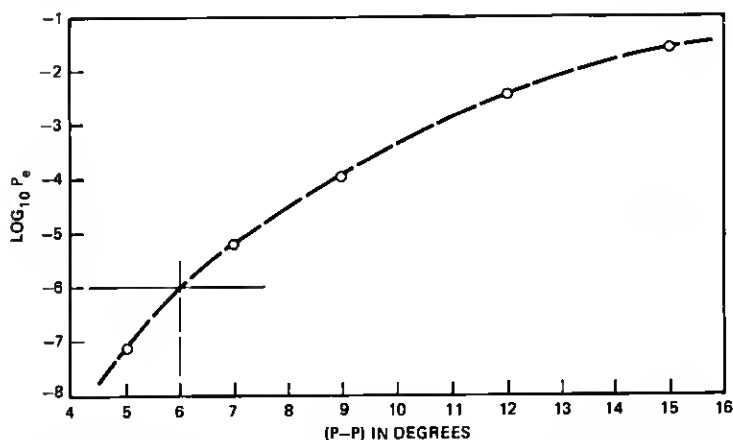


Fig. 3—The probability of error bound versus peak-to-peak phase jitter $f^2(t_0) \cdot \sigma_a^2 / \sigma^2 = 24$ dB, four-level system.

V. CONCLUSIONS

An upper bound of the error probability of a VSB-AM data system operated in the presence of additive Gaussian noise and phase jitter is presented in this correspondence. By restricting our attentions to these two parameters alone, it has been possible to calculate curves which can be used to estimate the accuracy required of a phase jitter recovery system.

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